Grade 11—Mathematics

The **academic expectations** addressed by "Simplifying Factorials" are

2.7   Students understand number concepts and use numbers appropriately and accurately.

2.8   Students understand various mathematical procedures and use them appropriately and accurately.

The **core content** assessed by this item includes

Number/Computation Skills

- Students are able to use operations on real numbers such as opposite, reciprocal, absolute value, power, root, logarithm, and factorial.

- Students should understand the complex number system.

Algebraic Ideas

- Students should understand variables and constants in expressions, equations, and inequalities.

Simplifying Factorials

The expression, 6!, is read "6 factorial" and means 6! = 6 x 5 x 4 x 3 x 2 x 1. Therefore, 6! = 720.

a. With this definition in mind, compute the value of each of the following numbers:

$$\frac{7!}{5!} \qquad \frac{9!}{6!}$$

b. Explain why the following equation is a true statement:

$$\frac{112!}{109!} = 112 \bullet 111 \bullet 110$$

c. Based on your work in **part b**, give a general expression that is equivalent to $\dfrac{n!}{(n-5)!}$

Score	Description
4	Student finds the correct values for part a. Student gives a complete explanation that shows the ability to eliminate like factors to determine expressions that are equivalent in part b. Student gives a correct equivalent expression for part c.
3	Student gives a complete explanation in part b. Student gives a correct equivalent expression for part c. Part a is incorrect. **OR** Student finds the correct values for part a. Student gives a correct equivalent expression for part c. Part b is incomplete but shows some correct reasoning. **OR** Student finds the correct values for part a. Student gives a complete explanation in part b. Part c is incorrect but shows some correct reasoning beyond substituting values for *n*.
2	Student finds the correct values for part a. Student shows some correct reasoning in part b and/or part c. **OR** Student gives a correct response for part b or part c. Part a is incorrect.
1	Student finds one or both correct values for part a; parts b and c are incorrect or missing. **OR** Student shows minimal understanding of the factorial operation. (Minimal response shows a simplification of an expanded factorial.)
0	Response is totally incorrect or irrelevant.
Blank	No response.

Note: At 4 level student must show parentheses on part c expression.

<u>CORRECT ANSWERS</u>

part a
7! = 7 x 6 x 5 x 4 x 3 x 2 x 1 = 5,040
5! = 5 x 4 x 3 x 2 x 1 = 120
5040 / 120 = 42

9! = 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1 = 362,880
6! = 6 x 5 x 4 x 3 x 2 x 1 = 720
362,880 / 720 = 504

part b
112! = 112 x 111 x 110 x 109 x 108 x 107 x…
109! = 109 x 108 x 107 x 106 x…

$$\frac{109 \times 108 \times 107 \times 106 \times ...}{109 \times 108 \times 107 \times 106 \times ...} = 1$$ Thus, $$\frac{112!}{109!} = 112 \text{ x } 111 \text{ x } 110 \text{ x } 1$$

part c
equivalent expression: $n(n–1)(n–2)(n–3)(n–4)$

Sample 4-Point Response of Student Work

Student Response

a) $\dfrac{7!}{5!} = \dfrac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1} = 42$

$\dfrac{9!}{6!} = \dfrac{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{6 \times 5 \times 4 \times 3 \times 2 \times 1} = 504$

b) $\dfrac{112!}{109!} = 112 \times 111 \times 110$ because 112! can also be written as 112 x 111 x 110 x 109!. Therefore, the 109!'s can cancel: 109!/109! is equal to 1. The equation looks like this:

$\dfrac{112!}{109!} = \dfrac{112 \times 111 \times 110 \times 109!}{109!} = 112 \times 111 \times 110$

c) $\dfrac{n!}{(n-5)!} = (n)\,(n-1)\,(n-2)\,(n-3)\,(n-4)$

Ex:

$\dfrac{9!}{(9-5)!} = \dfrac{9!}{4!} = \dfrac{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1}$

$= \quad 9 \times 8 \times 7 \times 6 \times 5$

$(n)\,(n-1)\,(n-2)\,(n-3)\,(n-4)$

← Student finds the correct values for part a.

← Student gives a complete explanation for part b that includes simplifying the fraction (i.e., $\dfrac{109!}{109!} = 1$).

← Student provides a correct equivalent expression (and an example to clarify the meaning) for part c.

Overall, the student shows a solid understanding of number concepts and mathematical procedures by fully responding to the prompt with complete and correct explanations.

Sample 4-Point Response of Student Work

Student Response

a) $\dfrac{7!}{5!} = 7 \times 6 = 42$

This is because the factorial of five is included in the factorial of 7. Since 5! is being divided the only values left are the numbers included in 7! that are not in 5! This is only 7 and 6. These multiply to equal 42.

$\dfrac{9!}{6!} = 7 \times 8 \times 9 = 504$

This problem works the same way, only 7, 8 and 9 are left, and their product equals 504.

b) $\dfrac{112!}{109!} = 112 \times 111 \times 110$ is true.

This is true because all values of 109 or less are canceled by the division of 109!

c) $\dfrac{n!}{(n-5)!} = (n-4)(n-3)(n-2)(n-1)(n)$

← Student finds the correct values for part a.

← Student gives a complete explanation for part b that includes simplifying the fraction (i.e., $\dfrac{109!}{109!} = 1$).

← Student provides a correct equivalent expression for part c.

Overall, the student shows a solid understanding of number concepts and mathematical procedures by fully responding to the prompt with complete and correct explanations.

Sample 3-Point Response of Student Work

Student Response

A. The value of $\frac{7!}{5!}$ is easily figured by multiplying $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$.

$$\frac{7!}{5!} = \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1} = \frac{5040}{720} = 42$$

The value of $\frac{9!}{6!}$ can be found the same way

$$\frac{9!}{6!} = \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{6 \times 5 \times 4 \times 3 \times 2 \times 1} = \frac{362880}{720} = 504$$

B. $\frac{112!}{109!} = 112 \cdot 111 \cdot 110$ is a true statement because if you multiply the top number of the fraction with the numbers below it until you get to the number 1 higher than the denominator the answer is the same as multiplying out the factorials.

C. $\frac{n!}{(n-5)!} = (N) \cdot (N-1) \cdot (N-2) \cdot (N-3) \cdot (N-4)$

← Student finds the correct values for part a.

← Student's explanation for part b is incomplete (i.e., does not clearly demonstrate or explain how to simplify the fraction) but shows some correct reasoning.

← Student provides a correct equivalent expression for part c.

Overall, the student shows an understanding of number concepts and mathematical procedures by responding correctly to most of the prompt.

Sample 2-Point Response of Student Work

Student Response

A) $\frac{5040}{720} = \frac{7!}{5!} = 42$

$\frac{9!}{6!} = \frac{362880}{720} = 504$

B) Because the 109 is 3 less than the 112, you need to only do the number in between the numerator and denominator, 112, 111, 110.

C) $\frac{n!}{(n-5)!}$ = N–5 · N–4 · N–3 · N–2 · N–1

← Student finds the correct values for part a.

← Student's explanation for part b is incomplete (i.e., does not demonstrate or explain how to simplify the fraction) but shows some correct reasoning.

← Student provides an incorrect expression for part c (i.e., missing the factor of *n* and neglecting to place parenthesis around the factors).

Overall, the student shows gaps in understanding of number concepts and mathematical procedures by responding correctly to only part of the prompt.

Sample 1-Point Response of Student Work

Student Response

A. $\dfrac{7!}{5!} = 7 \times 6 \times 5 = 42$

$\dfrac{9!}{6!} = 9 \times 8 \times 7 = 504$

B. $\dfrac{112!}{109!} = 112 \cdot 111 \cdot 110$

This statement is true because $\dfrac{112!}{109!}$ is defined by the numbers between 112 and 109. That's how the mathematic made it.

C. $\dfrac{n!}{(n-5)!}$ $\quad$ $\dfrac{6}{1}$ $\quad$ $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 = 720$

→ Student finds the correct values for part a.

→ Student's explanation for part b does not show sufficiently correct reasoning to earn a score of 2.

→ Student's response to part c is incorrect (i.e., shows an example, but does not provide a general equivalent which would show the ability to form an algebraic expression).

Overall, the student demonstrates limited understanding of number concepts and mathematical procedures by responding incorrectly or incompletely to most of the prompt.

The open-response item **"Simplifying Factorials"** is designed to assess students' ability to (1) understand and use instructions on a mathematical procedure, (2) simplify factorials, (3) determine and prove mathematical statements, and (4) write equivalent algebraic expressions. The instructional strategies below present ideas for helping students practice and master these concepts and skills.

Review the properties of complex numbers.

Remind students to verify their conclusions with a variety of methods such as expanding or simplifying algebraic expressions, using concrete examples, or substituting values for variables.

Teach students a variety of strategies for explaining a process. Remind them of the importance in explaining every step, using clear language, and using examples to clarify a statement.

Teach students to create and compare equivalent expressions.

Provide opportunities for students to work individually, in pairs, in small groups, and/or as a class to complete (with teacher support and guidance) any or all of the following activities:

- Explore and demonstrate understanding of reading instructions on mathematical procedures and put the instructions into practice.

- Prove statements true or false, using clear and complete evidence.

- Expand, simplify, and create algebraic expressions and equivalent expressions.

- Practice operations on real numbers such as reciprocal, power, root, and factorial.